EMPLOYING CONCRETE-REPRESENTATION-ABSTRACT APPROACH IN ENHANCING MATHEMATICS PERFORMANCE

Cristian Tenebro Camañan
Faculty, College of Education, Arts, and Sciences, Cor Jesu College
camaan.chrischan@gmail.com

ABSTRACT

This quasi-experimental research study aims to determine the effect of two teaching approaches—the concrete-representation-abstract approach and conventional approach—in enhancing the performance of students in Mathematics. Participants were grouped into a control group that was exposed to the conventional approach and experimental group that was exposed to the CRA approach. Pre-test and post-test of the two groups were gathered and analyzed using mean, paired sample t-test, independent sample t-test, and analysis of covariance (ANCOVA). Results revealed that there was no significant difference found between the pre-test mean scores of the experimental group and the control group. A significant difference was established between the pre-test and post-test mean scores of the students in both the control and experimental group. Lastly, there was a significant difference between the post-test mean scores of the control group and the experimental group while controlling the pre-test scores. It was also found out that the CRA approach had a higher mean score than the conventional teaching approach. Thus, the CRA approach found to be better than the conventional in enhancing students' mathematics performance.

Keywords: concrete-representation-abstract approach, traditional lecture approach, mathematics performance, quasi-experimental research, Philippines

INTRODUCTION

The poor performance in Mathematics subject has been a continuing problem that necessitates concrete approach and strategy to address. It is necessary to do because Mathematics achievement can be considered as a measure of success in academic advancement. Despite the importance of mathematics, it was found that many students face difficulties in learning, and teachers suffer in making students understand the subject (Hidayat & Prabawanto, 2018). These low performing students cannot identify and carry out routine and obvious procedures, and they are incapable of employing basic algorithms, formulas, procedures or conventions to solve problems and interpret results literally (Alzhanova-Ericsson, Bergman, and Dinnétz, 2017; Braza & Supapo, 2014; European Commission, 2011, 2013a, 2013c, 2014; OECD, 2014).

The teacher's capacity is one of the important factors that contributes to the quality of education in the classroom. It is for the teachers to modify and adjust their pedagogical skills to accelerate the learning process in the classroom (Behlol, Akbar, & Sehrish, 2018). Proper teaching methods characterize all good teaching, and priority may be given to improve the capacities and professional competencies of the teachers to apply appropriate teaching methods that may enhance the learning of students.

In trying to look at situations around the world, the Program for International Student Assessment (PISA) (2015) reported specifically in mathematics achievement, among the seventy-two (72) participating countries, that only 24 or about only 33% performs above average while the 67% lies below the mean score. No less than 30 of the 56 other countries that participated in the Program for International Student Assessment (PISA) math test had a larger percentage of students who scored at the international equivalent of the advanced level on our National Assessment of Educational Progress (NAEP) tests (Hanushek, Peterson & Woessmann, 2010). Evidently, the poor quality of mathematics education in South Africa can be traced to the low achievement levels reached by the students in standardized tests (Reddy et al., 2012). It was also found out that the African students in the international exams such as Trends in International Mathematics and Science Study (TIMSS) have consistently shown that students were not able to do less than the standardized and curricular expectations for their respective level (Sandefur, 2018).

Neighboring countries in Asia like Singapore, Japan, Chinese Taipei, Vietnam, and Korea are among the Asian countries that have only reached the above-average, respectively (PISA, 2015). In Thailand, the result of the PISA test in 2015 shows Thai students had much lower scores than their neighboring countries (Armstrong & Laksana, 2016). Thai students are underperforming their peers in several Asian countries as their scores were below the international average in Mathematic subjects (Mala, 2016).

In the Philippines, it was found out that its education system underperformed among peers in East Asia and the Pacific (Cordero, 2018). In the World Bank report (2018), it was noted that the Philippines, alongside Indonesia, Malaysia, and Thailand, scored below average in international exams under the Programme for International Student Assessment (PISA). In addition, among the 41 participant countries that participated in the PISA in 2011, Filipinos performed poorly in Mathematics (Nambatac, 2011). Moreover, it is reported that Filipino students have poor performance in Science and Mathematics subjects. In fact, the National Mean Percentage Score in Math in 2012 was only 48.90, which is described as below the national standard, and it is among the lowest in the five subjects in the National Achievement Test (NETRC, 2012).

Locally, the school year 2018-2019 first semester, students who enrolled in General Education 4 (Mathematics in the Modern World) under the College of Education, Arts, and Sciences (CEDAS) have a low result with respect to their academic performance on the said subject. In fact, 59.38% got a grade lower than 80, which is below the cut-off grade of the said program. In addition, there were only 34.37% from the class population who got grades between 80-90, and there were only four (4) students or 6.25% from the population who got a grade higher than 90. Thus, this shows a rising concern in the subject of mathematics to the aforementioned college.

Outside of the pedagogical debates in Mathematics, Martin (2013) pointed out the strategy in teaching Mathematics seems to prepare students as future Mathematicians, although very few of these students will reach the university level. Thus, one instructional approach, Concrete-Representational-Abstract (CRA,) is consistently used for supporting students especially those who have difficulties in mathematics (Flores, Hinton, Stroizer & Terry, 2014; Stroizer, Hinton, Flores, & Terry, 2015; Yakubova, Hughes, & Shinaberry, 2016). Previous studies stressed out also that the CRA approach found to be effective and beneficial to students. It was also found that students who were taught using the CRA learning approach scores higher than conventional or traditional learning (Putri, Misnarti, & Saptini, 2018; Salingay, & Tan, 2018; Hughes E., 2011). In fact, in the study conducted by Salingay & Tan (2018), the students under CRA significantly performed higher than those under the non-CRA. This was supported by the study of Calfoforo (2013) about multiple representation-based instructions, which also found out that the performance of the students exposed to multiple representation-based instructions is significantly higher than the control group.

The fact that there is a continuing problem on the poor performance of students in mathematics and the popularly acclaimed effectiveness of the Singaporean CRA approach in helping students perform better in mathematics is what prompted the researcher to conduct this study (Salingay & Tan, 2018). Specifically, the researcher would like to implement a quasi-experimental research design using the CRA approach to determine whether it is applicable and effective for Filipino students. The limited studies showing how effective the CRA approach in helping Filipino students learn mathematics is another reason that prompted the researcher to conduct this study.

Review of Related Literature

This review provides interrelated literature and studies about the two teaching approaches, namely Concrete-Representation-Abstract and Conventional, which give clarification and elaboration in the discussion of the nature of the study.

Concrete-Representation-Abstract Approach

The concrete-representational-abstract is an approach that combines the behaviorist and constructivist practices (Sealander, Johnson, Lockwood & Medina, 2012). This strategy is especially effective when used to teach individuals across grade levels and in many different topic areas in Mathematics (Hughes, Riccomini & Schneider, 2018). CRA uses demonstration, modeling, and guided practice followed by independent practice and immediate feedback, which are aspects commonly found in direct instruction. CRA also includes discovery-learning strategies involving representation to help students' transition between conceptual knowledge and procedural knowledge (Sealander, Johnson, Lockwood & Medina, 2012).

Recent research highlighting the CRA framework emphasizes the need to teach each phase to mastery (Flores, Hinton, Strozier, & Terry, 2014; Mancl, Miller, & Kennedy, 2012; Strickland & Maccini, 2013). Hence, the CRA sequence is an effective instructional model that increases conceptual, procedural, and declarative knowledge in mathematics (Mancl, Miller, & Kennedy, 2012). Teachers ought to start at the concrete level before moving to the representational level and, finally, the abstract level. The third part of the sequence, abstract thinking, will only be required if the information cannot be readily represented at the concrete or representational level. To achieve this outcome, teachers need to plan carefully and use innovative strategies in class.

The concrete understanding ought to be attempted first by using appropriate concrete objects. This uses well-planned instruction using physical manipulates, which allow students to become active participants in knowledge construction (Ünlü, 2018). When students use manipulatives to explore concepts, they are more engaged and motivated. They can participate in mathematical discourse, share their thinking, and reflect on their learning. This leads to increased achievement and a deeper understanding of the concepts studied (NCTM, 2010). Furthermore, the use of manipulatives increases the number of sensory inputs a student uses while learning the new concept, which improves the chances for a student to remember the procedural steps needed to solve the problem. The concrete phase develops conceptual learning using three-dimensional manipulatives (Witzel, Ferguson, & Mink, 2012). Hence, students will have an easier way of remembering the process of answering Mathematics problems.

On the other hand, representational understanding is achieved by using an appropriate drawing technique and, finally, appropriate strategies are used to assist learners in moving towards the abstract level of understanding of the concepts and symbols for a particular mathematical idea using explicit teaching. Other visual representations can take the form of a graphic organizer. Graphic organizers, in themselves, are search-supported intervention, depict the relationship between facts and concepts, helping a student to organize, and subsequently solve math problems (Strickland & Maccini, 2010; Strickland & Maccini, 2013). Hence, research supports the use of representational over pictorial graphics (van Garderen, Scheuermann, Poch, & Murray, 2018). This implies that the manipulations in the concrete and representational stages allow students to rationalize the conceptual mathematical procedures into logical steps and understandable definitions (Jones & Tiller, 2017). When students encounter a difficult mathematical problem, they are able to construct pictorial representations to assist in finding the solution (Witzel & Kiuhara, 2017). Teachers should use appropriate concrete manipulatives first. It is also important to ensure that learners acquire, retain, and master the mathematics skills at each stage of the instructional sequence (Hughes, Riccomini & Witzel, 2018). With that, students will be able to perform Mathematical procedures correctly.

Finally, the last phase of CRA is solving problems using numbers only, the abstract level. Prior to this level, students developed a conceptual understanding of numbers and operations using manipulatives, then pictures and drawings. Instruction at the abstract level focuses on completing tasks and solving problems using numbers only, developing procedural knowledge and fluency or declarative knowledge (Miller, Stringfellow, Kaffar, Ferreira, & Mancl, 2011). Strickland and Maccini (2013) asserted that generalizing the previously mastered conceptual understanding from the concrete and representational phases and applying that to the abstract phase is the most challenging aspect for students. Often students are supported in the transition between the representational and abstract phases through the application of a cognitive strategy (e.g., mnemonic or cue) that prompts students to remember the steps in the given mathematical process (Mancl, Miller & Kennedy, 2012). Miller, Stringfellow, Kaffar, Ferreira, and Mancl (2011) suggested that transition to the abstract phase is facilitated when students are exposed to Mathematical notation alongside presentations of Mathematics problems in concrete and representational formats.

To sum up, using the CRA instructional sequence will not guarantee accurate and complete mathematical understanding and success in any and every secondary Mathematics concept. However, there is a guarantee that the explicit sequence and multisensory approach of CRA provide flexibility when implemented in any Mathematical concepts. Resource room teachers may choose to introduce math topics and support general education abstract instruction using concrete and pictorial instruction before abstract instruction as a student preparation intervention. This approach was used to connect lessons and stages, which is necessary for learning targeted skills and comprehending associated concepts (Jones & Tiller, 2017; Witzel & Kiuhara, 2017). Using partial components of the Concrete-Representation-Abstract instructional method (i.e., abstract only, representational only, or representational and abstract only) did not demonstrate the significant gains in measures compared when the instruction uses the three methods (i.e., there is the abstract, representation and concrete). Therefore, it is a must that in using the CRA approach, these three methods should be done accordingly for better academic gains (Mancl, Miller, & Kennedy, 2012).

Conventional Teaching Approach

Teaching methods and approaches play a vital role in the teaching and learning process. There had been plenty of methods and approaches used. But, one such is the lecture method. The conventional or traditional teaching method is the oldest teaching method and still the most frequently used method of instruction inside the classroom throughout the world (McKeachie & Svinicki, 2014). The traditional conventional approach of teaching is teacher-centered with minimal or no active participation from the students. It has minimal or no integration of the subject, both horizontal and vertical (Kamran, Rehman, & Iqbal, 2011). The lecture method bases itself upon the transmissive teaching model- that is, knowledge is an object that can be transferred from the teacher to the learner. Practically, it implies a lecturer holding a lecture for a group of people ((Marmah, 2014). Moreover, teachers are given the full initiative to assist the students in obtaining the required knowledge. The instructor is the central focus of information transfer. As a result, students are passive information receivers because the instructor will just stand in the class and present information for students to learn (Marmah, 2014).

In line with this, there had been studies which pointed out that lecture is still widely accepted instructional method (Turner, 2015). In fact, in many developing countries, lecturing is the most dominant and traditional method of instruction (Ali, Shah, Amad, & Amad, 2012), and the majority of the Mathematics teachers follow the traditional methods to teach Mathematics (Nafees, 2011). In addition, Harris & Pamapaka (2016) stated that conventional lectures are effective in organizing and transmitting content knowledge. Ali (2011) further reports that responses from students exposed to a lecture format included their preference for the lecture because this method allowed the teacher to lead in a structured way as compared to the group work were a discussion among students lead to unnecessary chatter. Other researches also revealed that there are no differences in terms of learning outcomes between the lecture method and other teaching methods (Hafezimoghadam, Farahmand, Farsi, Zare, & Abbasi, 2013) which can be concluded that traditional lecture method is still an effective strategy in presenting a lesson, especially in mathematics. In fact, a study conducted by Benson, Orr, Biggers, Moss, Ohland, and Schiff (2010) provides evidence that students place greater emphasis on the lecture. Most of the students rated the lecture method as the best teaching method. Reasons given by the students included; teacher provides all knowledge related to the topic. It is a time-saving method, and students listen to the lecture attentively and take notes.

However, according to Kaur (2011), the traditional method is not for long-term learning because it can stifle learner's creativity. This approach only allows students to be dependent on their professors and, as a result avoiding responsibility in their own learning (Zachry, Nash, & Nolen, 2017). Pure lecture fails to give feedback to both the teacher and the learners. Lectures cannot keep students' attention for a long time or for the whole lesson. Information is easily forgotten if taught through the lecture method. Lectures assume that all learners have the same learning styles (Ayodele & Fatoba, 2017). Lectures cannot teach motor skills, influence attitudes and values, teach application, analysis, synthesis, or evaluation (Finley, 2016).

It was also discovered from the study of Shaari, Yusoff, Ghazari, Osman, and Dzahir (2014) that students preferred interactive teaching techniques over traditional lecture methods because traditional lectures can only encourage a shallow approach to learning (Waldrop, 2015). Despite two opposing ideas towards the traditional lecture method, there is enough evidence from research studies indicating that most of the university teachers are not familiar with modern teaching methods and that traditional lectures are still the most popular instructional method in the universities (Adib-Hajbaghery & Aghajani, 2011). However, the findings from the study of Sakala (2012) showed that teachers had been exposed to a variety of teaching methods and techniques during their initial teacher training. It was further reported in this same study that down the line as they practiced their career, teachers resorted to using the lecture technique more than any other teaching techniques.

Henceforth, having these pieces of literature, it is confirmed that CRA is an approach we needed in actively learning math concepts. It should always be remembered that to promote active learning by students and to have meaningful learning to occur, and students must actively engage with the to-be-learned subject-matter through activities such as discussion, hands-on activities, and problem-solving (Freeman et al., 2014). Moreover, Maphosa, Zikhali, Chauraya, and Madznire (2013) also concluded that students perceived lecture with discussion as an effective pedagogical approach confirmed the importance of student involvement in learning as opposed to wholly lecturer dominated teaching sessions. This proves the fact that traditional lectures are perceived as effective and are appreciated by many if they are modified. In this case, it is not a lecture technique per se, but an eclectic approach to teaching in which a lecture technique is only a component. Other than that, the teacher's personality also weighs great deals in the effectiveness of the lecture method. Evidence shows that students' mathematics learning is influenced by the teaching they experience at school (Sullivan et al., 2015). Students' understanding of mathematics, their ability to solve problems, and their confidence in and their disposition towards mathematics are all shaped by the teaching they encounter in school, and the success of the students depends most of all on the quality of teaching offered by the teacher.

Concrete-Representation-Abstract Approach and Mathematics Performance

Predicting students' performance at school is considered crucial for students, educators, policymakers, and stakeholders; and therefore, the factors that can be linked to academic achievement have been investigated by many research studies. However, it is accepted that the role of the teacher is pivotal in the Mathematics achievement of a student. At the same time, it is also very crucial how the teacher provides meaningful representations and situations to the students (Kang, 2012). Effective mathematics instruction is an appropriate balance of the participation of the teacher and his/her learners (Kilpatrick, Swafford, & Findell, 2001; NMAP, 2008). Instead, the balance of instruction is determined by the needs of the students and the nature of the instructional content. Well-designed curricula, in association with competent teachers, aid student learning (NCTM, 2010).

In connection, in Singapore's mathematics curriculum CRA stands out in helping the students improve their academic performance (Kaur, Tay, Toh, Leong & Lee, 2018; Men, Ismail, & Abidin, M, 2018; Salingay & Tan, 2018; Toh, & Kaur, 2019). Researchers have shown that the use of the CRA sequence of instruction has been very effective and beneficial to learners who struggle with understanding mathematical concepts and procedures performance (Flores, 2009; Putri, 2015; Salingay & Tan, 2018; Witzel, Riccomini, & Schneider, 2008;). As a whole, it is pointed out that in order to have a positive result towards students' Mathematics performance, the proper sequencing of the identified stage must be observed. Even a single stage that was not properly worked out and even a single mistake upon implementation of this approach can affect the whole performance of students in learning and will not demonstrate the significant gains in mathematics (Mancl, Miller, & Kennedy, 2012).

Theoretical Framework

This study was anchored on two theories namely Activity Theory based on the work Vygotsky and his student Leont'ev from their studies of cultural-historical psychology in the 1920s (Verenika, 2001); and second is Jerome Bruner's "Theory of Representation of Knowledge" (1966).

Activity Theory pointed out that humans are not separated from their social environment, and their actions are mediated by tools. This theory uses the whole work activity as the unit of analysis, where the activity is broken into the analytical components of subject, tool, and object, where the subject is the person being studied, the object is the intended activity, and the tool is the mediating device by which the action is executed (Hasan, 1998). The use of tools is a means for the accumulation and transmission of social knowledge. It influences the nature, not only of external behavior but also of the mental functioning of individuals.

In this particular study, CRA employs a series of activities using mediating tools such as manipulates and technologies. This was employed to provide students with scaffolding. It is posited that by providing manipulatives, it would effectively influence the mental functioning of the students towards grasping difficult topics in mathematics. The researcher, acting as the teacher, presented different stages for the mediating tools. At first, the students created a manipulative and used it during the discussion to represent the abstract ideas in mathematics. Then, a certain technology was used to give an illustration or picture of the topic and present it to the students.

Moreover, this study was also anchored on Bruner's Theory of Representation of Knowledge. This explains that the acquisition of knowledge has three stages, namely enactive, iconic, and symbolic (Leong, Ho & Cheng, 2015; Teng, 2014; Wong, 2015). According to Bruner (1964), conceptual learning begins with experience from actions undertaken through the support of manipulatives (enactive), then it was subsequently translated into images of the experience formed (iconic). With an accumulation of enactments and their corresponding iconic representations, links are formed to connect some of the representations into a collective structure. The criteria for selection into the collective structure is governed by a certain rule derived from organizing common attributes found embedded in those qualified representations. Eventually, this rule ascends above the enactive and iconic representations to stand exclusively by itself and is denoted by a symbol (Bruner & Kenney, 1965).

The first stage is the enactive or concrete stage; the learners learn about the world or can understand abstract ideas through actions on physical objects or manipulatives. Thus, mediating tools are highly needed, especially in an activity. It is suggested that manipulating familiar and confidence-inspiring entities, whether they are physical (blocks, sticks, counters, rods) as Bruner suggested, or meta-physical (numerals as numbers, letters as variables or as generalities, familiar diagrams, screen manipulable objects, and many more).

Second is the iconic stage or representation. Tools are still needed since the students' learning can be obtained through the use of models or pictures of the given topic because the knowledge is still stored primarily in the form of visual images. The learner can use mental images to stand for certain objects or events.

Third is symbolic or the abstraction stage, mediating tools are not needed. This last stage of representation of knowledge stressed out that learners have developed the ability to think in abstract terms. This uses symbol systems such as language and mathematical notations to encode knowledge.

Conceptual Framework

The conceptual framework of the study follows the interrelatedness of the approaches in teaching Mathematics—the Concrete-Representational-Abstract Approach and the Conventional Approach and the Mathematics achievement of the students.

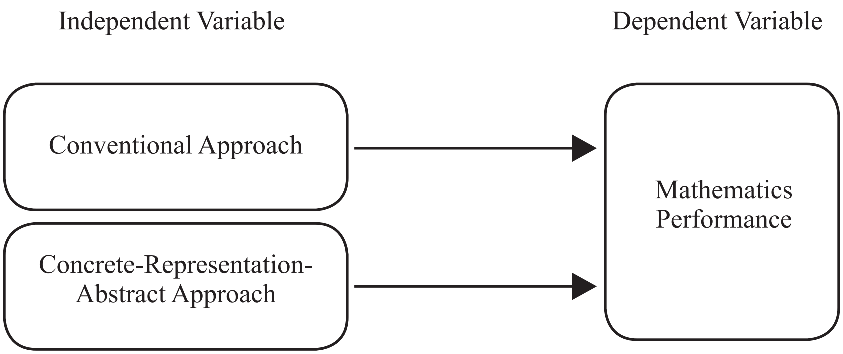

Figure 1. Conceptual Framework

The figure represents the flow of the study, which illustrates the effectiveness of the Concrete-Representation-Abstract approach and the conventional method, which are the independent variables. The performance of the students was based on their pre-test and post-test mean scores as the dependent variable. The control group was given the conventional approach, whereas the experimental group was given the Concrete-Representation-Abstract Approach.

Objective of the Study

This study aimed to determine the effect of CRA in enhancing the performance of students in Mathematics. Specifically, the study sought to investigate if there is a significant difference between the pre-test mean scores of experimental and control groups. Then it also investigated if there a significant difference between the pre-test mean score and the post-test mean score of students in the control group and experimental group. Finally, it also examined if there a significant difference in the post-test mean score of control and experimental groups while controlling the pre-test scores.

METHOD

This study used a quasi-experimental design. This design is suitable to be employed when it is not feasible to randomly design participants of control and experimental groups for comparison (Cook, Campbell & Shadish, 2002). Further, this employed pretest-posttest non-equivalent design to compare the effectiveness of the two teaching methods—Conventional Approach and Concrete-Representation-Abstract Approach using the mean scores. The subjects were the intact groups of college freshmen students who were officially enrolled in Gen Education 4 subject in a naturally accumulated classroom setting.

Below is the Nonequivalent Control Group Design presented by Campbell and Stanly (1966) which was employed in this study:

$$\begin{array}{ccc} O_1 & X & O_2 \\ \hline O_3 & & O_4 \end{array}$$

where:

X	=	*Experimental Treatment*
O1	=	*Pretest of the Experimental Group*
O2	=	*Posttest of the Experimental Group*
O3	=	*Pretest of the Control Group*
O4	=	*Posttest of the Control Group*
------	=	*Non-randomized selection of the students*

This design used a pre-test and post-test tool for both the control and experimental groups, identical in all aspects except that there was a non-random assignment of treatment. Finally, according to Shaughnessy, Zechmeister, and Zechmeister (2014), the quasi-experimental is appropriate to employ if the purpose is to analyze the difference of the pre-test mean scores between the experimental group and control group and the post-test mean scores of the groups. The respondents of this study were the freshmen college students who were officially enrolled in General Education 4 (Mathematics in the Modern World) subject for the school year 2018-2019 second semester. Moreover, this study utilized simple random sampling, specifically the toss coin technique. The toss coin technique was used to avoid bias in choosing who among the two blocks will be the experimental group and the control group. The proponents of this study utilized a 35-item teacher-made multiple-choice questionnaire as an instrument to measure students' performance in Mathematics. This questionnaire underwent a series of tests before it was used. First, it was subjected for validity test, which was validated by three qualified validators, namely Dr. Jonald Fenecios, Dr. Joy Picar, and Dr. Ronald Decano was asked to assure the validity of the said self-made questionnaire. The validation sheet was given to the validators using the format used by Cor Jesu College Graduate School. Results from content validity reveal that the overall mean score is 4.58 and is interpreted excellent. This result indicates that the instrument is excellent in terms of its clarity of directions and items, presentation and organization of items, the suitability of items, adequateness of items per category indicator, attainment of purpose, and objectivity. After integrating the minor corrections on terminologies, the instrument was then finalized.

Table 1. Results and Interpretation of Content Validity

Area	Mean Score	Interpretation	Overall Mean Score	Interpretation
Clarity of Directions and Items	4.33	Very Satisfactorily	4.72	Excellent
Presentation and Organization of Items	4.67	Excellent		
Suitability of Items	4.67	Excellent		
Adequateness of items per category indicator	4.67	Excellent		
Attainment of Purpose	5	Excellent		
Objectivity	4.67	Excellent		
Scale and Evaluation Rating Scale	5	Excellent		

Second, it had gone through the reliability test through pilot testing. The result of pilot testing was subjected to statistical analysis using Cronbach's alpha. Results of reliability statistics on table 2 show that Cronbach's alpha where all above 0.65, which implies that all items tested passed the test of reliability. The abovementioned questionnaire was composed of 35 items consisting of items taken from the five identified learning competencies, namely: Euler circuit, Euler Path, Hamiltonian circuit, Hamiltonian circuit, Hamiltonian path, and weighted graph using the nearest neighbor algorithm and cheapest link algorithm. Exactly the same questionnaire was used to both control and experimental groups. Thus, the mathematics performance of the two groups was measured through students' scores from both pre-test and post-test given.

Table 2. Results and Interpretation of Reliability Test

Factor	Cronbach's Alpha	No. of Items	Interpretation
Euler Circuit	.801	8	Reliable
Euler Graph	.758	7	Reliable
Hamiltonian Circuit	.825	8	Reliable
Hamiltonian Path	.714	6	Reliable
Weighted Graph	.680	6	Reliable

The following scales were utilized to interpret the mean scores of the respondent in Mathematics using a grading point system in the college department, and the interpretation was based on the Department of Education K-12 curriculum.

Table 3. Table of Interpretation for the Students' Mean Scores in the Pre-test and Post-test

Numerical Equivalent	Description	Interpretation
31.51-35.00	Advanced	The student, at this level, exceeds the core requirements in terms of knowledge, skills, and understanding and can transfer them automatically and flexibly through authentic performance tasks.
27.51-31.50	Proficient	The student, at this level, has developed fundamental knowledge, skills, and core understandings and can transfer them independently through authentic performance tasks.
23.51-27.50	Approaching Proficiency	The student at this level has developed the fundamental knowledge, skills, and core understandings with a little guidance from the teacher and/or with some assistance from peers who can transfer these understandings through authentic performance tasks.
21-23.50	Developing	The student, at this level, possesses the minimum knowledge and skills and core understandings but needs help throughout the performance of the authentic tasks.
21-below	Beginning	The student, at this level, struggles with his/her understanding, prerequisite, and fundamental knowledge and/or skills that have not been acquired or developed adequately to aid something.

The researcher sought permission from the dean of the Graduate School of the Cor Jesu College to conduct the study. When the approval was granted, the researcher then asked permission from the dean of the College of Education, Arts and Sciences (CEDAS) then to the program heads where the study was conducted. When the permission was given to conduct the study, the researcher administered the pre-test to the subject Gen. Ed 4 (Mathematics in the Modern World) in both the experimental and control group based on their class schedule on January 11. After conducting the pre-test, the researcher, then, started the intervention on January 14. The results of the pre-test were properly recorded and retained. Then, after using the two different approaches to two different groups, the researcher gave specific instructions about the nature of the post-test and how to perform it. Afterward, the scores from the pre-test and post-test were collated, tallied, tabulated, analyzed, and interpreted by the researcher with the aid of the statistician. In analyzing the data, mean was used to identify the mean score of Mathematics performance of the students in the experimental and control group. Then, a paired sample t-test was employed to find the difference in the pre-test and post-test of the students in the control group, as well as to determine the significant difference in the pre-test and post-test of the experimental group.

Independent t-test was used to find the significant difference between the pre-test mean scores of experimental and control groups. Finally, analysis of covariance as employed to find if there is a significant difference in the post-test mean scores of control and experimental groups while controlling the pre-test mean scores.

Results

This section explains the presentation, analysis, and interpretation of the data gathered using the most appropriate statistical tool. It starts by discussing the significant difference in the pre-test scores of the students in Mathematics performance of experimental and control groups. The presentation continued by discussing the significant difference between the pre-test mean score, and the post-test means score of students in the control group. Then, next to it was the discussion on the significant difference between the pre-test and post-test scores of students in experimental groups. The chapter ends by discussing the significant difference in the post-test mean score of control and experimental groups while controlling the pre-test scores.

Significant Difference in the Pre-test Scores of the Students in Mathematics Performance of Experimental and Control Groups

In this study, the researcher tried to determine if there is a significant difference between the pre-test means scores of experimental and control groups. In order to answer the research problem, a T-test for Independent Sampled was used. Results reveal that the students in the control group show a higher level of mathematics performance with a mean score of 14.84 as compared to those students who are in the experimental group with a mean score of 13.5.

Table 6. Significant Difference in the Pre-test Scores of the Students in Mathematics Performance of Experimental and Control Group

Groups	Mean Score	Significance (2-tailed)	Decision	Interpretation
Control	14.84	.090	Fail to reject the null hypothesis	Not Significant
Experimental	13.5			

However, when attempted to determine whether such difference can be considered as significant, the results reveal a significance value of 0.090, which is found to be higher than the 0.05 level of significance set for this study. This implies that the difference in the mean scores of the two groups of students is found insignificantly different. Therefore, the null hypothesis has failed to be rejected. It can be argued, therefore, that before the treatment was employed, students of the two groups have most likely the same level of mathematics performance, which is at the beginning stage. They have difficulty in understanding the mathematics of graphs, especially the five identified subject matter.

Significant Difference Between the Pre-test Scores and Post-test Scores of Students in Control Group

The study aimed to determine whether the conventional teaching method is effective in enhancing mathematics performance. The level of mean scores of mathematics performance was determined before and after the intervention or treatment. Mean scores were calculated, and a Paired-sample t-test was employed in order to establish whether there is a significant difference in the performance of students in mathematics before and after the treatment is used.

Table 7. Significant Difference in the Pre-test and Post-test Scores of the Students in Mathematics Performance of Control Group

Tests	Mean Score	Significance (2-tailed)	Decision	Interpretation
Pre-test	14.84	.000	Reject the null hypothesis	Significant
Post-test	19.32			

Results showed that the level of mathematics performance of the students before the treatment was at 14.84 or interpreted as the beginning. Then, after the students were exposed to the treatment or intervention, the level of their mathematics performance rose to 19.32 for an increase of about 4.47, but still belongs to the beginning stage. Moreover, the difference in the mean scores was found to be significant, as manifested by the result (2-tailed) value of 0.000, which is found to be below the 0.05 level of significance set in this study. Hence, this still indicates that the level of improvement of the mathematics performance of the students after being exposed to the treatment is found to be considerable or significant and that the treatment can be considered as a helpful tool in enhancing Mathematics performance. Thus, the null hypothesis is rejected.

Significant Difference Between Pre-test Scores and Post-test Scores of Students in Experimental Group

Another aim of this study was to determine whether the Concrete-Representation-Abstract method is effective in enhancing Mathematics performance. The level of mean scores of Mathematics performance was determined before and after the intervention or treatment. Mean scores were calculated, and a Paired-sample t-test was employed in order to establish whether there was a significant difference in the mathematics performance of students in the experimental group before and after the treatment was employed.

Table 8. Significant Difference in the Pre-test and Post-test Scores of the Students in Mathematics Performance of Experimental Group

Tests	Mean Score	Significance (2-tailed)	Decision	Interpretation
Pre-test	13.5	.007	Reject the null hypothesis	Significant
Post-test	22.45			

Results show that the level of Mathematics performance of the students before the application of the Concrete-Representation-Abstract Approach was at 13.5. Then, after the students were exposed to CRA for a period of time, the level of mathematics performance rose to 22.45 for an increase of about 8.95.

Interestingly, the difference in the mean scores was found to be significant, as manifested by the result (2-tailed) value of 0.007, which was found to be below the 0.05 level of significance set in this study. This indicates that the level of improvement of the mathematics performance of the students after using the CRA Approach is found to be significant and that the CRA Approach can be considered as an effective tool in enhancing mathematics performance where students' performance in Mathematics at this time is at developing stage. Thus, the null hypothesis is rejected.

Significant Difference Between the Post Test Mean Scores of the Students in the Control and Experimental Groups While Controlling the Pre-test Scores

This study tried to determine if there is a significant difference in the post-test scores of control and experimental groups while controlling the pre-test scores. Analysis of Covariance (ANCOVA) was utilized.

Table 9. Significant Difference in the Post-test Scores of Control and Experimental Groups while Controlling the Pre-test Scores

Source	Sum of Squares	df	Mean Square	F-value	Sig.
Group	167.639	1	167.639	12.019	.001
Pre-test (covariate)	144.393	1	144.393	10.352	.002
Error	934.536	67	13.948		
Total	30613.00	70			

Results reveal that the mean score of the control group in the post-test was 19.32, while the experimental group was 22.45, which is 3.94 higher than the control group. Interestingly, the difference in the mean gain scores was significant, as manifested by the F-value of 12.019 and supported by a sig (2-tailed) value of 0.001, which is found below the 0.05 level of significance set in this study. This indicates that there is a significant difference in the post-test scores of control and experimental groups while controlling the pre-test scores. This means that there is a greater improvement in the mathematics performance of the students in the experimental group than that of the control group. This upholds that the employment of the Concrete-Representation-Abstract approach is more effective than the Conventional Teaching Approach in enhancing students' Mathematics performance.

Discussion

Examining the effect of the CRA approach in enhancing the performance of students in Mathematics could be beneficial for both administrators, teachers, students, and future researchers. The results of this study could help the administrators which would provide with new ideas which they can recommend for the further implementation of the curriculum, for Math Teachers to use this creative teaching strategy for their subjects so as to aid them in improving the students' academic performance, especially in mathematics, for the students to develop higher-order thinking skills which makes them capable of understanding different content in Mathematics.

As tried to determine if there is a significant difference between the pre-test means scores of experimental and control groups, no significant difference was found. The result only proves that low performance in mathematics exists wherein none from the group have reached the proficient level. These low performing students, perhaps, cannot identify and carry out routine, distinct procedures, and they are incapable of employing basic algorithms, formulas, procedures, or conventions to solve problems and interpret results literally (European Commission, 2013c, 2014; OECD, 2014c).

A significant difference was found between the pre-test scores and post-test scores of the control group. The result connects to the investigation done by Covill (2011). It has been found out in her study that students considered the conventional lecture method to be more comprehensive. They were reported to have great involvement in the teaching-learning process and becoming more involved in independent thinking and problem-solving. Also, according to Harris & Pampaka (2016), the lecture-style of teaching remains the preferred way of students in higher education very much. Ali (2011) further reported that responses from students exposed to a lecture format enclosed their preference for the lecture since it allows the teacher to guide them during a structured manner as compared to the group work where discussion among students result in reserve chatter.

With regard to the significant difference between pre-test scores and post-test scores of students in the experimental group, a significant difference was also found. The result of this study supports the claim of previous researches that students who are identified to have learning mathematics difficulty learn better when CRA approach is used since the test result shows that the students of the control group, the one who uses the CRA approach, has higher test result (Flores, Hinton, Stroizer & Terry, 2014; Stroizer, Hinton, Flores, & Terry, 2015; Yakubova, Hughes, & Shinaberry, 2016) It is in line with the study of Putri (2015) which concludes that the implementation of CRA approach in mathematics learning can enhance the ability of mathematical representation and spatial sense.

As to the significant difference between the post-test mean scores of the students in the control and experimental groups, while controlling the pre-test scores, a significant difference was also found. The result of this study confirms the findings of Salingay and Tan (2018) confirmed that students under the CRA Approach have significantly performed higher than those under the non-CRA. Calfoforo (2013) also supports, thus indicating that students exposed to Multiple Representation-Based Instructions are significantly higher than the control group. Carbonneau, Marley, and Selig (2013) also emphasize that using manipulatives in mathematics instruction produces a small- to medium-sized effect on student learning when compared with instruction that uses abstract symbols alone.

Conclusions

Findings reveal that after the traditional lecture approach was employed, students' Mathematics performance found to have increased. Thus, the effectiveness of such an approach had been established. Then, after the CRA approach was utilized, students' Mathematics performance also improved. Hence, the effectiveness of the CRA had been established. Thus, students exposed to the CRA approach perform significantly better than those students in the traditional lecture approach. Since the results concluded that Concrete-Representation-Abstract Approach was found to be better than the conventional or traditional lecture approach in enhancing students' mathematics performance.

Recommendations

In light of the findings and conclusions of the study, the following recommendations were offered: School administration should strongly consider the application of CRA as a teaching approach to teach mathematics from basic education to higher education. Also, the school administration should conduct a series of training to enhance the skills of mathematics teachers on how to effectively employ CRA in the classroom. Moreover, the least acquired Mathematics competencies of students should be regularly evaluated so that results can be used in providing materials needed to implement the CRA approach by the teachers. Lastly, similar experimental studies should be conducted involving more students with longer time duration to establish the effectiveness and functionality of the CRA approach fully.

REFERENCES

Adib-Hajbaghery, M., & Aghajani, M. (2011). Traditional lectures, Socratic method and student lectures: Which one do the students prefer?

Ali, H. (2011). A comparison of cooperative learning and traditional lecture methods in the project management department of a tertiary level institution in Trinidad and Tobago. *Caribbean Teaching Scholar*, 1(1): 49-64.

Alzhanova-Ericsson, A. T., Bergman, C. & Dinnetz, P. (2017). Lecture attendance is a pivotal factor for improving prospective teachers' academic performance in Teaching and Learning Mathematics. *Journal of Furhter and Higher Education, 41(1),* 1-15.

American Institute for Research. (2016). Mathematatics Education. Retrieved on January 15, 2019 from goo.gl/uQC2FK

Ampadu, E., & Danso, A. (2018). Constructivism in Mathematics Classrooms: Listening to Ghanaian Teachers' and Students' Views. *Africa Education Review*, 1-23.

Armstrong, N., & Laksana, S. (2016). Internationalization of Higher Education: Case Studies of Thailand and Malaysia. Scholar, 8(1), 102.

Ayodele, M. O., & Fatoba, J. O. (2017). Effects of Small Group Learning Instruction on Atittudes and Performance of Basic Science Students in Ekiti Sate, Nigeria. *European Journal of Social Sciences Studies*.

Behlol, M. G., Akbar, R. A., & Sehrish, H. (2018). Effectiveness of Problem Solving Method in Teaching Mathematics at Elementary Level. *Bulletin of Education and Research*, *40*(1), 231-244.

Benson, L. C., Orr, M. K., Biggers, S. B., Moss, W. F., Ohland, M. W., & Schiff, S. D. (2010). Student-centered active, cooperative learning in engineering. International *Journal of Engineering Education*, 26(5), 1097-1110.

Braza, M & Supapao, S. (2014). *Effective Solutions in the implementation of the K to 12 Mathematics Curriculum.* West Visayas State University. College of Education. La Paz, Ilo- ilo City, Philippines.

Bruce, C.D. (2007). Student interaction in the math classroom: Stealing ideas or building understanding. What Works? Research into Practice. Retrieved from http://www.edu.gov.on.ca/eng/literacynumeracy/in spire/research/Bruce.pdf

Bruner, J. S. (1964). The course of cognitive growth. *American psychologist*, *19*(1), 1.

Bruner, J. S., & Kenney, H. J. (1965). Representation and mathematics learning. *Monographs of the Society for Research in Child Development*, *30*(1), 50-59.

Carbonneau, K. J., Marley, S. C., & Selig, J. P. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal of Educational Psychology*, *105*(2), 380.

Calfoforo, A. V. (2013). Multiple representation-based instruction: Effects on the performance and attitudes of students in high school Algebra. *Unpublished Masters' Thesis. Central Mindanao University*

Cook, T. D., Campbell, D. T., & Shadish, W. (2002). *Experimental and quasi-experimental designs for generalized causal inference*. Boston, MA: Houghton Mifflin.

Cordero, T. (2018). PHL education system underperforms in East Asia Pacific —WB report. Retrieved from GMA News Online.

Covill, A. E. (2011). College Students' Perceptions of the Traditional Lecture Method. *College Student Journal*, *45*(1).

Enríquez, J. A. V., de Oliveira, A. M. P., & Valencia, H. G. (2018). What Mathematics Teachers Say about the Teaching Strategies in the Implementation of Tasks. *English Language Teaching*, *11*(1), 65-79.

European Commission (2011). Mathematics education in Europe: Common challenges and national policies. Brussels: Education, Audiovisual and Cultural Executive Agency, Eurydice.

European Commission (2013b). PISA 2012: EU performance and first inferences regarding education and training policies in Europe. Brussels.

European Commission (2013c). Thematic Working Group on mathematics, science and technology: Addressing low achievement in mathematics and science. Brussels.

European Commission (2014). *Study to prepare the commission report on policies for tackling low achievement in basic skills. Luxembourg*: Publication Office of the European Union

Fahiminezhad, A., Mozafari, S., Sabaghiyanrad, L., Esmaeili, R. M. (2012). The Effect of Traditional and Integration Methods of Teaching on The Amount of Learning Math and Sport Performance of First Grade of Elementary Students, *Euro. J. Exp. Bio.,* 2012, 2 (5):1646-1653.

Finley, J. P. (2016). Exploring Meteorology Education in Community College: Lecture-based Instruction and Dialogue-based Group Learning (Doctoral dissertation, Lesley University).

Flores, M. M. (2009). Teaching subtraction with regrouping to students experiencing difficulty in mathematics. *Preventing School Failure: Alternative Education for Children and Youth*, *53*(3), 145-152.

Flores, M. M., Hinton, V. M., Strozier, S. D., & Terry, S. L. (2014). Using the concrete-representational-abstract sequence and the strategic instruction model to teach computation to students with autism spectrum disorders and developmental disabilities. *Education and Training in Autism and Developmental Disabilities*, 547-554.

Flores, M. M., Hinton, V., & Strozier, S. D. (2014). Teaching subtraction and multiplication with regrouping using the concrete-representational-abstract sequence and strategic instruction model. *Learning Disabilities Research & Practice*, *29*(2), 75-88.

Freeman, S., Eddy, S. L., McDonough, M., Smith, M. K., Okoroafor, N., Jordt, H., &

Wenderoth, M. P. (2014). Active learning increases student performance in science, engineering, and mathematics. *Proceedings of the National Academy of Sciences*, *111*(23), 8410-8415.

Hafezimoghadam P, Farahmand S, Farsi D, Zare M, Abbasi S (2013). A comparative study of lecture and discussion methods in the education of basic life support and advanced cardiovascular life support for medical students. Turk. J. *Emerg Med*.; 13(2):59-63.

Hanushek, E., Peterson, P., & Woessmann, L. (2010). How well does each state do at producing high-achieving students? Retrieved on March 5, 2019 from goo.gl/KRkeZB

Harris, D., & Pampaka, M. (2016). 'They [the lecturers] have to get through a certain amount in an hour': first year students' problems with service mathematics lectures. *Teaching Mathematics and Its Applications: International Journal of the IMA*, *35*(3), 144-158.

Hasan, H. (1998). *Activity Theory: a Basis for the Contextual study of Information Systems in Organizations. In H. Hasan, E. Gould & P. N. Hyland (Eds.), Information Systems and Activity Theory: Tools in Context* (pp. 19-38). Wollongong: University of Wollongong Press.

Hidayat, W., & Prabawanto, S. (2018, January). Improving students' creative mathematical reasoning ability students through adversity quotient and argument driven inquiry learning. In *Journal of Physics: Conference Series* (Vol. 948, No. 1, p. 012005). IOP Publishing.

Hughes, E. M. (2011). Focus on.

Hughes, E. M., Riccomini, P. J., & Witzel, B. (2018). Using Concrete-Representational-Abstract Sequence to Teach Fractions to Middle School Students with Mathematics Difficulties. *JEBPS Vol N2*, *16*(2), 171.

Jones, J. P., & Tiller, M. (2017). Using concrete manipulatives in mathematical instruction. *Dimensions of Early Childhood*, *45*(1), 18-23.

Kamran A, Rehman R, Iqbal A. Importance of clinically oriented problem solving tutorials (COPST) in teaching of physiology. *Rawal Med J. 2011*;36(3):232-6.

Kaur, B., Tay, E. G., Toh, T. L., Leong, Y. H., & Lee, N. H. (2018). A study of school mathematics curriculum enacted by competent teachers in Singapore secondary schools. *Mathematics Education Research Journal*, *30*(1), 103-116.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). ADDING IT.

Kim, T. K. (2015). T test as a parametric statistic. *Korean journal of anesthesiology*, *68*(6), 540-546.

Leong, Y. H., Ho, W., K., & Cheng, L., P. (2015). Concrete-Pictorial-Abstract: Surveying its origins and charting its future. *The Mathematics Educator,* 16(1), 1-9.

Mala. D (2016). Thai education fails the test while Singapore and Vietnam excel. Why?: special report: Vietnam sparks surprise after moving up sharply in PISA test rankings. *Bangkok post*. Retrieved from http://www.bangkokpost.com/learning/advanced/1163240/thai-education-fails-the-test while-singapore-and-vietnam-excel-why-

Mancl, D. B., Miller, S. P., & Kennedy, M. (2012). Using the concrete-representational-abstract sequence with integrated strategy instruction to teach subtraction with regrouping to students with learning disabilities. *Learning Disabilities Research & Practice*, *27*(4), 152-166.

Mann, E. L., Chamberlin, S. A., & Graefe, A. K. (2017). The prominence of affect in creativity: Expanding the conception of creativity in mathematical problem solving. In *Creativity and Giftedness* (pp. 57-73). Springer, Cham.

Marbach-Ad, G., Seal, O., & Sokolove, P. (2001). Student attitudes and recommendations on active learning. *Journal of College Science Teaching*, *30*(7), 434.

Marmah, A. A. (2014). Students' perception about the lecture as a method of teaching in tertiary institutions. views of students from college of technology education, Kumasi (COLTEK). *International Journal of Education and Research*, *2*(6), 601-612.

Martin, J. (2013). *A necessary renewal of Mathematics Education*. Retrieved on January 6, 2014 from http://goo.gl/jsBqy2

Mercer, C. D., & Miller, S. P. (1992). Teaching students with learning problems in math to acquire, understand, and apply basic math facts. *Remedial and Special Education*, *13*(3), 19-35.

Men, O. L., Ismail, Z., & Abidin, M. (2018, December). Using Maths Model Method in Solving Pre-Algebraic Problems Among Year Five Students. In *2018 IEEE International Conference on Teaching, Assessment, and Learning for Engineering (TALE)* (pp. 222-227). IEEE.

Miller, S. P., Stringfellow, J. L., Kaffar, B. J., Ferreira, D., & Mancl, D. B. (2011). Developing computation competence among students who struggle with mathematics. *Teaching Exceptional Children,* 44(2), 38– 46. Retrieved from http://eric.ed.gov/?id=EJ945988

Maphosa, C., Zikhali, J., Chauraya, E. and Madzanire, D. (2013). "Students' Perceptions of the Utility of Pedagogical Approaches Used by Lecturers in Selected Faculties at a Zimbabwean University. "*Journal Social Science,* 6 (2): 113-122.

Moust, J. H., Berkel, H. V., & Schmidt, H. G. (2005). Signs of erosion: Reflections on three decades of problem-based learning at Maastricht University. *Higher education*, *50*(4), 665-683.

Nafees, M. (2011). An experimental study on the effectiveness of problem-based versus lecture-based instructional strategy on achievement, retention and problem solving capabilities in secondary school general science students. *PhD Unpublished Thesis, International Islamic University, Islamabad*.

Nambatac, (2011). Mathematica Communication, Conceptual Understanding and Student's Attitude in Mathematics. MAT Degree. *Department of Mathematics University, Nebrasca- Lincoln.*

National Achievement Test (NAT) 2012. National Education Training and Research Institute (NETRC).

National Mathematics Advisory Panel. (2008). Foundations for success: The final report of the National Mathematics Advisory Panel. Washington, DC: U.S. Department of Education

NCTM, (2010). National Council for Teacher in Mathematics Report.

OECD, (2014). PISA 2012 Technical Report.

Putri, H.E. (2015). The Influence of Concrete-Pictorial-Abstract Approach to the Mathematical (CPA) Approach to the Mathematical Representation Ability Achievement of the Pre-service Teachers at Elementary School. *International Journal of Education and Research*. Vol. 3 No. 6 June 2015.

Putri, H. E., Misnarti, M., & Saptini, R. D. (2018). Influence of Concrete-Pictorial-Abstract (CPA) Approach Towards the Enhancement of Mathematical Connection Ability of Elementary School Students. *Edu Humaniora*, *10*(2), 6171.

PISA, (2015). Excellence and Equity in Education, summarizes student performance in PISA 2015, and examines inclusiveness and fairness in participating education systems, http://dx.doi.org/10.1787/9789264266490-en.

Reddy, V., Prinsloo, C., Arends, F., Visser, M., Winnaar, L., Feza, N., & Ngema, M. (2012). Highlights from TIMSS 2011: The South African perspective.

Ross, A., & Willson, V. L. (2017). *Paired Samples T-Test. In Basic and Advanced Statistical Tests* (pp. 17-19). SensePublishers, Rotterdam.

Sadler, B. J. (2004). How important is student participation in teaching philosophy? *Teaching philosophy*, 27(3): 251-267.

Sakala, J. (2012). Factors Contributing to Excess use of the Lecture Method among High School Teachers in Selected Schools of Kabwe and Kalulushi Districts: Lessons for Educational Administrators. *Unpublished Master's Dissertation. University of Zambia*.

Salingay, N., & Tan, D. (2018). Concrete-Pictorial-Abstract Approach On Students' Attitude And Performance In Mathematics. *International Journal of Scientific & Technology Research*, *7*(5).

Sandefur, J. (2018). Internationally comparable mathematics scores for fourteen African countries. *Economics of Education Review*, *62*, 267-286.

Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly*, *23*(2), 139-159.

Schoen, H. L., Fey, J. T., Hirsch, C. R., & Coxford, A. F. (1999). Issues and options in the math wars. *Phi Delta Kappan*, 80: 444–453.

Sealander, K. A., Johnson, G.R., Lockwood, A. B., & Medina, C. M. (2012). Concrete-semiconcrete-abstract (CSA) instruction: A decision rule for improving instructional efficacy. *Assessment for Effective Intervention, 30,* 53-65.

Shaari, A. S., Yusoff, N. M., Ghazali, I. M., Osman, R. H., & Dzahir, N. F. M. (2014). The relationship between lecturers' teaching style and students' academic engagement. *Procedia-Social and Behavioral Sciences*, *118*, 10-20.

Shaughnessy, J.J., Zechmeister, E.B. and Zechmeister, J.S. 2014. Research Methods in Psychology. Online Retrieved from http://www. mhhe.com/socscience/psychology/shaugh/index.html (accessed 25/11/14)

Strickland, T. K., & Maccini, P. (2010). Strategies for teaching algebra to students with learning disabilities: Making research to practice connections. *Intervention in School and Clinic*, *46*(1), 38-45.

Strickland, T. K., & Maccini, P. (2013). The Effects of the Concrete–Representational–Abstract Integration Strategy on the Ability of Students with Learning Disabilities to Multiply Linear Expressions Within Area Problems. *Remedial and Special Education*, *34*(3), 142-153.

Snell, Y. S. L. S. (1999). Interactive lecturing: strategies for increasing participation inlarge group presentations. *Medical Teacher*, *21*(1), 37-42.

Sullivan, P., Askew, M., Cheeseman, J., Clarke, D., Mornane, A., Roche, A., & Walker, N. (2015). Supporting teachers in structuring mathematics lessons involving challenging tasks. *Journal of Mathematics Teacher Education*, *18*(2), 123-140.

Tanujaya, B., Prahmana, R. C., & Mumu, J. (2017). Mathematics instruction, problems, challenges, and opportunities: A case study in Manokwari regency, Indonesia. *World Transactions on Engineering and Technology Education*, *15*(3), 287-291.

Tharayil, S., Borrego, M., Prince, M., Nguyen, K. A., Shekhar, P., Finelli, C. J., & Waters, C. (2018). Strategies to mitigate student resistance to active learning. *International Journal of STEM Education*, *5*(1), 7.

Teng, A. (2014*). Ex-teacher Kho Teck Hong solved Singapore's maths problem: Ex-teacher led team that created model method for teaching, learning*. The Straits Times, p.B7

Toh, T. L., & Kaur, B. (2019). Low Attainers and Learning of Mathematics. In *Mathematics Education in Singapore* (pp. 287-311). Springer, Singapore.

Turner, Y. (2015). Last orders for the lecture theatre? Exploring blended learning approaches and accessibility for full-time international students. *The International Journal of Management Education*, *13*(2), 163-169.

Ünlü, M. (2018). Effect of Micro-Teaching Practices with Concrete Models on Pre-service Mathematics Teachers' Self-efficacy Beliefs about Using Concrete Models. *Universal Journal of Educational Research*, *6*(1), 68-82.

van Garderen, D., Scheuermann, A., Poch, A., & Murray, M. M. (2018). Visual representation in mathematics: Special Education teachers' knowledge and emphasis for instruction. *Teacher Education and Special Education*, *41*(1), 7-23.

Verenikina, I. (2001). *Cultural-Historical Psychology and Activity Theory in Everyday Practice. In H. Hasan, E. Gould, P. Larkin & L. Vrazalic (Eds.), Information Systems and Activity Theory*: Volume 2 Theory and Practice (pp. 23-38). Wollongong: University of Wollongong Press

Witzel, B. S., & Kiuhara, S. A. (2017). *Overcoming Mathematics Difficulties using CRA Interventions.*

Witzel, B., Ferguson, C., & Mink, D. (2012). Strategies for helping preschool through grade 3 children develop math skills. *Young Children*, 67(3), 89-94.

Wong, K. Y. (2015). *Effective mathematics lessons through an eclectic Singapore approach*: Yearbook 2015. Association of Mathematical Educators. Singapore; World Scientific Publishing.

World Bank. World Development Report 2018

Yakubova, G., Hughes, E. M., & Hornberger, E. (2015). Video-based intervention in teaching fraction problem-solving to students with autism spectrum disorder. *Journal of Autism and Developmental Disorders*, 45: 2865–2875. doi:10.1007/s10803-015- 2449-y.

Yang Li-niang, Deng Jun. (2005). *"Seminar: Training graduate students scientific research ability of the effective way",* China's geological education, vol.3, pp. 5-7, 2005.

Zachry, A. H., Nash, B. H., & Nolen, A. (2017). Traditional lectures and team-based learning in an occupational therapy program: a survey of student perceptions. *The Open Journal of Occupational Therapy*, *5*(2), 6.